RiskAMP Reference Manual

RiskAMP Version 3

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Contents

Reference Cell

In order to most effectively utilize the calculation and modeling abilities of Excel, the RiskAMP add-in stores all simulation results within the spreadsheet, so that subsequent functions can analyze the simulation results and generate a statistical analysis.

For each iteration of a probability simulation, the entire spreadsheet is recalculated. In a typical simulation, this can happen hundreds or thousands of times. For the subsequent analysis, we need to reach back over each iteration and find values from the calculated spreadsheet.

It's not practical to store the contents of every cell in the spreadsheet for each iteration. This would use up a lot of memory and add to the processing time. Therefore, the RiskAMP add-in uses a concept of *Reference Cells*. Basically, the idea is that a simulation function – one of the RiskAMP statistical analysis functions – must refer to a cell in the spreadsheet before the data from that cell is stored.

For each reference cell – each cell that is referred to by a simulation function – the probability simulation engine stores the value of that cell at each iteration of a simulation. That means that every value of that cell can later be retrieved, and the cell can be used as the basis for later analysis.

One of the most useful features of the reference cell concept is that once the cell data is stored, it can be used for any simulation analysis function – even functions that were not originally intended to be used. This allows users of the add-in to explore data and statistics after the simulation has completed, changing parameters and reviewing the model, withing re-running the simulation. Of course the simulation can be re-run at any time, to generate new data or to modify the simulation model.

Many of the simulation analysis and statistics functions in the RiskAMP add-in use the concept of *Reference Cells*. During a probability simulation, the entire spreadsheet is recalculated. Any cells containing random variables (using the RiskAMP random functions or built-in Excel random functions) will be populated with new values.

In addition, the reference cell can be any cell in a spreadsheet – not just one of the RiskAMP random distribution functions. That means that any value, or function result, can form the basis of a probability simulation. You are not limited to the random functions the add-in provides.

Simulation Functions

Simulation functions are used to analyze the results of a probability simulation, after the simulation has run. In general, simulation functions take one or more *Reference Cells* as parameters (in addition to any additional parameters); see the section on Reference Cells for more information on the concept.

Simulation functions can return both statistical values (i.e. analyses of the complete simulation) and discrete results (i.e. results from a particular iteration of the simulation).

SimulationAverage

SimulationCorrelation

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SimulationValue

SimulationVariance

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Average

Spreadsheet Function:	SimulationAverage
	This function is also available as *SimulationMean*
Parameters:	ref : the reference cell
Example:	= SIMULATIONAVERAGE(A1)

The SimulationAverage function returns the mean, or average value, of the reference cell over all iterations of the simulation.

Correlation

Spreadsheet Function:	SimulationCorrelation
Parameters:	ref 1 : the first reference cell
	ref 2 : the second reference cell
Example:	= SIMULATIONCORRELATION(A1, A2)

The SimulationCorrelation function returns the observed correlation between any two spreadsheet cells over all iterations of the most recently run simulation.

Covariance

Spreadsheet Function:	SimulationCovariance
Parameters:	ref 1 : the first reference cell
	ref 2 : the second reference cell
Example:	= SIMULATIONCOVARIANCE(A1, A2)

The SimulationCovariance function returns the observed covariance between any two spreadsheet cells over all iterations of the most recently run simulation.

Interval

Spreadsheet Function: SimulationInterval

Parameters: ref : the reference cell

min : the minimum value of the interval *(optional)*

max : the maximum value of the interval *(optional)*

Example: = SIMULATIONINTERVAL(A1, , 1)

The SimulationInterval function returns the probability (expressed as a percent) that over all iterations of the simulation, the observed value of the reference cell falls between the given minimum and maximum values.

If the minimum value is omitted, the function will return the probability that the value lies below the given maximum; similarly, if the maximum value is omitted, the function will return the probability that the value lies above the given minimum.

This function is used to determine the probability, after running a simulation, that a particular value will be reached or achieved. For example, in a project planning model, the function can be used to determine the likelihood – using a probability simulation – that the project will be completed on schedule, or within certain tolerances (one month, two months, etc).

Kurtosis

Spreadsheet Function: SimulationKurtosis

Parameters: ref : the reference cell

Example: = SIMULATIONINTERVAL(A1)

The SimulationKurtosis function returns the kurtosis of the reference cell, over all iterations of the most recently run simulation. Kurtosis is the degree of peakedness of a distribution, relative to a normal distribution. Perfectly normally-distributed data will have kurtosis of 3.

Maximum

Spreadsheet Function: SimulationMax

Parameters: ref : the reference cell

Example: = SIMULATIONMAX(A1)

The SimulationMax function returns the maximum observed value of the reference cell, over all iterations of the most recently run simulation.

Median

Spreadsheet Function: SimulationMedian

Parameters: ref : the reference cell

Example: = SIMULATIONMEDIAN(A1)

The SimulationMedian function returns the median value of the reference cell, over all iterations of the most recently run simulation.

Minimum

Spreadsheet Function: SimulationMin

Parameters: ref : the reference cell

Example: = SIMULATIONMIN(A1)

The SimulationMax function returns the minimum observed value of the reference cell, over all iterations of the most recently run simulation.

Mode

Spreadsheet Function: SimulationMode

Parameters: ref : the reference cell

Example: = SIMULATIONMODE(A1)

The SimulationMedian function returns the mode (most commonly observed value) of the reference cell, over all iterations of the most recently run simulation. This function does not return a true mode, but a synthetic mode derived by placing results into "buckets" (much like creating a histogram).

Percentile

Spreadsheet Function: SimulationPercentile

Parameters: ref : the reference cell

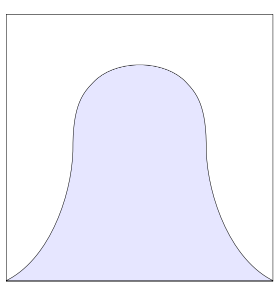
percentile : the percentile value to retrieve

Example: = SIMULATIONPERCENTILE(A1, 0.10)

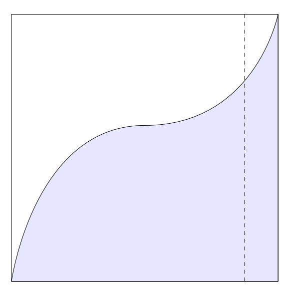
The SimulationPercentile function returns the expected value of the reference cell corresponding to the given percentile over all iterations of the simulation. A percentile value of 50% will return the mean value of the reference cell; 0% will return the minimum value of the reference cell; and 100% will return the maximum value of the reference cell.

This is one of the most useful functions in analyzing simulation data, but its use can be confusing at first. An understanding of how this function works can be helpful in utilizing it in spreadsheet models.

In order to determine the value at a given probability, the function first sorts all observed values of the reference cell over all iterations of the simulation in ascending order (low to high). It then returns the value corresponding to the percentile – e.g., in a simulation of 100 iterations, the 90th percentile corresponds to the 90th-highest value. Because the values have been sorted, we can say that in this simulation, 90% of the observed values fall at or below the returned value (and correspondingly, 10% of all values are at or higher than the returned value).



Normally-distributed values returned from a random function



The same values sorted in ascending order, with the 90th percentile value marked

In this manner the function can be used to describe confidence levels. In an investment return scenario, for example, the 10th percentile can be used to determine the minimum expected return, in all but the worst 10% of cases. This corresponds to a one-way confidence level[1] of 90%; that is, we are 90% confident that in a simulation, we will see a return equal to or better than the given value.

1 Two-way confidence levels are also commonly used to analyze data. A two-way confidence level is derived by taking the values at the center, rather than one side of the data. For example, a two-way confidence level of 90% is determined by finding the values at the 5th percentile and the 95th percentile. We know that 90% of the values lie between these two points; therefore, the 5th and 95th percentiles represent the low and high values of a 90% two-way confidence interval.

Skewness

Spreadsheet Function: SimulationSkewness

Parameters: ref : the reference cell

Example: = SIMULATIONSKEWNESS(A1)

The SimulationSkewness function returns the skewness of the reference cell over all iterations of the most recently run simulation. Skewness refers to the degree of asymmetry of a distribution. The normal distribution should be symmetric about the mean, with a skewness value of 0.

Standard Deviation

Spreadsheet Function: SimulationStandardDeviation

Parameters: ref : the reference cell

Example: = SIMULATIONSTANDARDDEVIATION(A1)

The SimulationStandardDeviation function returns the standard deviation of the reference cell over all iterations of the most recently run simulation.

Trials

Spreadsheet Function: SimulationTrials

Parameters:

Example: = SIMULATIONTRIALS( )

The SimulationTrials function returns the number of trials, or iterations, used in the most recently run simulation. This function can be used as the base for custom analysis of data or reporting; it is often used as the divisor in some analytic method.

Value

Spreadsheet Function: SimulationValue

Parameters: ref : the reference cell

index : the iteration number to retreive

Example: = SIMULATIONVALUE(A1, 1)

The SimulationValue function returns the observed value of a particular cell at a given iteration (the index) of the most recent simulation. For any cell in a spreadsheet, the value at each iteration can be retrieved using this method.

Variance

Spreadsheet Function: SimulationVariance

Parameters: ref : the reference cell

Example: = SIMULATIONVARIANCE(A1)

The SimulationVariance function returns the variance of the reference cell over all iterations of the most recently run simulation.

Sorted Simulation Index

Spreadsheet Function: SortedSimulationIndex

Parameters: ref : the reference cell

value : the target value to retrieve

Example: = SORTEDSIMULATIONINDEX(A1, 100)

This function is unique in that it returns an iteration index, rather than an observed or derived value. The output of the function, an index, is intended for passing to the SimulationValue function (which returns the value of a reference cell at a particular iteration of the simulation). This function can be used to determine the value of an input cell at iterations of the simulation in which some other output cell reached a particular value.

Continuous Distributions

Generally speaking, continuous distributions return the probability that an event will occur or not occur, based on input parameters particular to the distribution. The basic forms of most distributions return probability values between 0 and 1, although many of the functions contained in the add-in include parameters that affect the scale and offset of the returned values.

The continuous distribution functions contained in the add-in follow. See the description of the distribution for more information on the add-in functions.

ArcSineValue

BetaValue

CauchyValue

ChiSquareValue

CosineValue

DoubleLogValue

ErlangValue

ExponentialValue

FRatioValue

GammaValue

LaplaceValue

LogarithmicValue

LogisticValue

LognormalValue

NormalValue

ParabolicValue

ParetoValue

Pearson5Value

Pearson6Value

PowerValue

RayleighValue

StudentTValue

TriangularValue

UniformValue

WeibullValue

Arcsine

Spreadsheet Function: ArcsineValue

Parameters: xmin : minimum value of the returned random variable

xmax : maximum value of the returned random variable

Example: = ARCSINEVALUE(0, 1)

Density:
$$f(x) = \begin{cases} \dfrac{1}{\pi\sqrt{x(1-x)}} & 0 \leqslant x \leqslant 1 \\ 0 & \textit{otherwise} \end{cases}$$

Distribution:
$$F(x) = \begin{cases} 0 & x < 0 \\ \dfrac{2}{\pi}\sin^{-1}(\sqrt{x}) & 0 \leqslant x \leqslant 1 \\ 1 & x > 1 \end{cases}$$

Beta

Spreadsheet Function: BetaValue

Parameters: v : first shape parameter v > 0

w : second shape parameter w > 0

Example: = BETAVALUE(1, 5)

Density:

$$f(x)=\begin{cases} \dfrac{x^{v-1}(1-x)^{w-1}}{\mathrm{B}(v,w)} & 0\leqslant x\leqslant 1 \\ 0 & otherwise \end{cases}$$

where $\mathrm{B}(v,w)$ is the beta function $\mathrm{B}(v,w)\equiv\int_0^1 t^{v-1}(1-t)^{w-1}\,dt$

Distribution:

$$F(x)=\begin{cases} \mathrm{B}_x(v,w)/\mathrm{B}(v,w) & 0\leqslant x\leqslant 1 \\ 0 & otherwise \end{cases}$$

where $\mathrm{B}_x(v,w)$ is the incomplete beta function $\mathrm{B}_x(v,w)\equiv\int_0^x t^{v-1}(1-t)^{w-1}\,dt$

Cauchy / Lorentz

Spreadsheet Function: CauchyValue

Parameters: a : location parameter

b : scale parameter

Example: = CAUCHYVALUE(0, 1)

Density: $$f(x)=\frac{1}{\pi b}\left[1+\left(\frac{x-a}{b}\right)^2\right]^{-1} \quad -\infty<x<\infty$$

Distribution: $$F(x)=\frac{1}{2}+\frac{1}{\pi}\tan^{-1}\left(\frac{x-a}{b}\right) \quad -\infty<x<\infty$$

Chi-Square

Spreadsheet Function: ChiSquareValue

Parameters: v : degrees of freedom v ≥ 1

Example: = CHISQUAREVALUE(1)

Density:

$$f(x) = \begin{cases} \dfrac{x^{(v/2)-1} e^{-x/2}}{2^{v/2}\Gamma(v/2)} & \text{if } x > 0 \\ 0 & \textit{otherwise} \end{cases}$$

where $\Gamma(z)$ is the gamma function $\Gamma(z) \equiv \int_0^{\infty} t^{z-1} e^{-t}\, dt$

Distribution:

$$\begin{cases} F(x) = \dfrac{1}{2^{v/2}\Gamma(v/2)} \int_0^{x} t^{(v/2)-1} e^{-t/2}\, dt & \text{if } x > 0 \\ 0 & \textit{otherwise} \end{cases}$$

Cosine

Spreadsheet Function: CosineValue

Parameters: xmin : minimum value of the returned random variable

xmax : maximum value of the returned random variable

Example: = COSINEVALUE(0, 1)

Density:
$$f(x) = \begin{cases} \dfrac{1}{2b}\cos\left(\dfrac{x-a}{b}\right) & x_{min} \leqslant x \leqslant x_{max} \\ 0 & otherwise \end{cases}$$

Distribution:
$$F(x) = \begin{cases} 0 & x < x_{min} \\ \dfrac{1}{2}\left[1+\sin\left(\dfrac{x-a}{b}\right)\right] & x_{min} \leqslant x \leqslant x_{max} \\ 1 & x > x_{max} \end{cases}$$

Double Log

Spreadsheet Function: DoubleLogValue

Parameters: xmin : minimum value of the returned random variable

xmax : maximum value of the returned random variable

Example: = DOUBLELOGVALUE(0, 1)

Density:

$$f(x) = \begin{cases} -\frac{1}{2b}\ln\left(\frac{|x-a|}{b}\right) & x_{min} \leqslant x \leqslant x_{max} \\ 0 & otherwise \end{cases}$$

Distribution:

$$F(x) = \begin{cases} \frac{1}{2} - \left(\frac{|x-a|}{2b}\right)\left[1 - \ln\left(\frac{|x-a|}{b}\right)\right] & x_{min} \leqslant x \leqslant a \\ \frac{1}{2} + \left(\frac{|x-a|}{2b}\right)\left[1 - \ln\left(\frac{|x-a|}{b}\right)\right] & a \leqslant x \leqslant x_{max} \end{cases}$$

Erlang

Spreadsheet Function: ErlangValue

Parameters: b : scale parameter b > 0

c : shape parameter c, a positive integer

Example: = ERLANGVALUE(1, 2)

Density:

$$f(x)=\left\{\begin{array}{cc} \dfrac{(x/b)^{c-1}e^{-x/b}}{b(c-1)!} & x\geqslant 0 \\ 0 & otherwise \end{array}\right\}$$

Distribution:

$$F(x)=\left\{\begin{array}{cc} 1-e^{-x/b}\sum_{i=0}^{c-1}\dfrac{(x/b)^i}{i!} & x\geqslant 0 \\ 0 & otherwise \end{array}\right\}$$

Exponential

Spreadsheet Function: ExponentialValue

Parameters: a : location parameter a, any real number

b : scale parameter b > 0

Example: = EXPONENTIALVALUE(0, 1)

Density: $$f(x) = \left\{ \begin{array}{cc} \frac{1}{b} e^{-(x-a)/b} & x \geqslant a \\ 0 & otherwise \end{array} \right\}$$

Distribution: $$F(x) = \left\{ \begin{array}{cc} 1 - e^{-(x-a)/b} & x \geqslant a \\ 0 & otherwise \end{array} \right\}$$

Extreme Value

Spreadsheet Function: ExtremeValue

Parameters: a : location parameter a, any real number

b : scale parameter b > 0

Example: = EXTREMEVALUE(0, 1)

Density: $f(x)=\frac{1}{b}e^{(x-a)/b}\exp\left[-e^{(x-a)/b}\right] \quad -\infty<x<\infty$

Distribution: $F(x)=1-\exp\left[-e^{(x-a)/b}\right] \quad -\infty<x<\infty$

F Ratio

Spreadsheet Function: FRatioValue

Parameters: v : shape parameter 1 (degrees of freedom), a positive integer

w : shape parameter 2 (degrees of freedom), a positive integer

Example: = FRATIOVALUE(4, 4)

Density:
$$f(x)=\begin{cases} \dfrac{\Gamma[(v+w)/2]}{\Gamma(v/2)\Gamma(w/2)} \dfrac{(v/w)^{v/2} x^{(v-2)/2}}{(1+xv/w)^{(v+w)/2}} & x \geqslant 0 \\ 0 & otherwise \end{cases}$$

where $\Gamma(z)$ is the gamma function $\Gamma(z) \equiv \int_0^\infty t^{z-1} e^{-t} dt$

Distribution: No closed form, in general.

Gamma

Spreadsheet Function: GammaValue

Parameters: a : location parameter

b : scale parameter b > 0

c: shape parameter c > 0

Example: = GAMMAVALUE(0, 1, 2)

Density:

$$f(x) = \begin{cases} \frac{1}{\Gamma(c)} b^{-c} (x-a)^{c-1} e^{-(x-a)/b} & x > a \\ 0 & otherwise \end{cases}$$

where $\Gamma(z)$ is the gamma function $\Gamma(z) \equiv \int_0^\infty t^{z-1} e^{-t} dt$

If n is an integer, $\Gamma(n) = (n-1)!$

Distribution: No closed form, in general. However, if c is a positive integer, then

$$F(x) = \begin{cases} 1 - e^{-(x-a)/b} \sum_{k=0}^{c-1} \frac{1}{k!} \left(\frac{x-a}{b} \right)^k & x > a \\ 0 & otherwise \end{cases}$$

Laplace (Double Exponential)

Spreadsheet Function: LaplaceValue

Parameters: a : location parameter a, any real number

b : scale parameter b > 0

Example: = LAPLACEVALUE(0, 1)

Density: $$f(x)=\frac{1}{2b}\exp\left(-\frac{|(x-a)|}{b}\right) \quad -\infty<x<\infty$$

Distribution: $$F(x)=\begin{cases} \frac{1}{2}e^{(x-a)/b} & x\leqslant a \\ 1-\frac{1}{2}e^{-(x-a)/b} & x\geqslant a \end{cases}$$

Logarithmic

Spreadsheet Function: LogarithmicValue

Parameters: xmin : minimum value of the returned random variable

xmax : maximum value of the returned random variable

Example: = LOGARITHMICVALUE(0, 1)

Density:

$$f(x) = \begin{Bmatrix} -\frac{1}{b}\ln\left(\frac{x-a}{b}\right) & x_{min} \leqslant x \leqslant x_{max} \\ 0 & otherwise \end{Bmatrix}$$

Distribution:

$$F(x) = \begin{Bmatrix} 0 & x < x_{min} \\ \left(\frac{x-a}{b}\right)\left[1 - \ln\left(\frac{x-a}{b}\right)\right] & x_{min} \leqslant x \leqslant x_{max} \\ 1 & x > x_{max} \end{Bmatrix}$$

Logistic

Spreadsheet Function: LogisticValue

Parameters: a : location parameter a, any real number

b : scale parameter b > 0

Example: = LOGISTICVALUE(0, 1)

Density: $$f(x)=\frac{1}{b}\frac{e^{(x-a)/b}}{\left[1+e^{(x-a)/b}\right]^2} \quad -\infty<x<\infty$$

Distribution: $$F(x)=\frac{1}{1+e^{-(x-a)/b}} \quad -\infty<x<\infty$$

Lognormal

Spreadsheet Function: LognormalValue

Parameters: mu (μ) : mean of the distribution (scale parameter)

sigma (σ) : standard deviation σ > 0 (shape parameter)

a : location parameter a, any real number

Example: = LOGNORMALVALUE(0, 1, 0)

Density:

$$f(x) = \begin{cases} \dfrac{1}{\sqrt{2\pi}\sigma(x-a)} \exp\left[\dfrac{-\left[\ln(x-a)-\mu^2\right]}{2\sigma^2}\right] & x>a \\ 0 & otherwise \end{cases}$$

Distribution:

$$F(x) = \begin{cases} \dfrac{1}{2}\left\{1+\mathrm{erf}\left[\dfrac{\ln(x-a)-\mu}{\sqrt{2}\sigma}\right]\right\} & x>a \\ 0 & otherwise \end{cases}$$

Normal (Gaussian)

Spreadsheet Function: NormalValue

Parameters: mu (μ) : mean of the distribution (scale parameter)

sigma (σ) : standard deviation σ > 0 (shape parameter)

Example: = NORMALVALUE(0, 1)

Density: $f(x)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right]$

Distribution: $F(x)=\frac{1}{2}\left[1+\operatorname{erf}\left(\frac{x-\mu}{\sqrt{2}\sigma}\right)\right]$

Parabolic

Spreadsheet Function: ParabolicValue

Parameters: xmin : minimum value of the returned random variable

xmax : maximum value of the returned random variable

Example: = PARABOLICVALUE(0, 1)

Density: $f(x)=\frac{3}{4b}\left[1-\left(\frac{x-a}{b}\right)^2\right] \quad x_{min}\leqslant x\leqslant x_{max}$

Distribution: $F(x)=\frac{(a+2b-x)(x-a+b)^2}{4b^3} \quad x_{min}\leqslant x\leqslant x_{max}$

Pareto

Spreadsheet Function: ParetoValue

Parameters: c : shape parameter c > 0

Example: = PARETOVALUE(1)

Density: $$f(x) = \begin{cases} cx^{-c-1} & x \geq 1 \\ 0 & otherwise \end{cases}$$

Distribution: $$F(x) = \begin{cases} 1 - x^{-c} & x \geq 1 \\ 0 & otherwise \end{cases}$$

Pearson's Type 5 (Inverted Gamma)

Spreadsheet Function: Pearson5Value

Parameters: b : scale parameter b > 0

c : shape parameter c > 0

Example: = PEARSON5VALUE(1, 2)

Density:
$$f(x) = \begin{cases} \dfrac{x^{-(c+1)} e^{-b/x}}{b^{-c}\Gamma(c)} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

where $\Gamma(z)$ is the gamma function $\Gamma(z) \equiv \int_0^{\infty} t^{z-1} e^{-t} dt$

Distribution:
$$F(x) = \begin{cases} \dfrac{\Gamma(c, b/x)}{\Gamma(c)} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

where $\Gamma(a, z)$ is the incomplete gamma function $\Gamma(a, z) \equiv \int_z^{\infty} t^{a-1} e^{-t} dt$

Pearson's Type 6

Spreadsheet Function: Pearson6Value

Parameters:
b : scale parameter b > 0
v : shape parameter v > 0
w : second shape parameter w > 0

Example: = PEARSON6VALUE(1, 2, 4)

Density:

$$f(x)=\left\{\begin{array}{cc} \dfrac{(x/b)^{v-1}}{b\,\mathrm{B}(v,w)\left[1+(x/b)\right]^{v+w}} & \text{if } x>0 \\ 0 & \textit{otherwise} \end{array}\right\}$$

where $\mathrm{B}(v,w)$ is the beta function $\mathrm{B}(v,w)\equiv\int_0^1 t^{v-1}(1-t)^{w-1}\,dt$

Distribution:

$$F(x)=\left\{\begin{array}{cc} F_{\mathrm{B}}\left(\dfrac{x}{x+b}\right) & \text{if } x>0 \\ 0 & \textit{otherwise} \end{array}\right\}$$

where $F_{\mathrm{B}}(x)$ is the distribution function of a $\mathrm{B}(v,w)$ random variable

Power

Spreadsheet Function: PowerValue

Parameters: c : shape parameter c > 0

Example: = POWERVALUE(1)

Density: $f(x) = cx^{c-1} \quad 0 \leqslant x \leqslant 1$

Distribution: $F(x) = x^{c} \quad 0 \leqslant x \leqslant 1$

Rayleigh

Spreadsheet Function: RayleighValue

Parameters: a : location parameter a, any real number

b : scale parameter b > 0

Example: = RAYLEIGHVALUE(0, 1)

Density:

$$f(x) = \begin{cases} \dfrac{2}{x-a}\left(\dfrac{x-a}{b}\right)^2 \exp\left[-\left(\dfrac{x-a}{b}\right)^2\right] & x \geqslant a \\ 0 & otherwise \end{cases}$$

Distribution:

$$F(x) = \begin{cases} 1-\exp\left[-\left(\dfrac{x-a}{b}\right)^2\right] & x \geqslant a \\ 0 & otherwise \end{cases}$$

Student's t

Spreadsheet Function: StudentTValue

Parameters: v : degrees of freedom v, a positive integer

Example: = STUDENTTVALUE(1)

Density:
$$f(x) = \frac{\Gamma\left[(v+1)/2\right]}{\sqrt{\pi v}\,\Gamma(v/2)} \left(1 + \frac{x^2}{v}\right)^{-(v+1)/2} \qquad -\infty < x < \infty$$

where $\Gamma(z)$ is the gamma function $\Gamma(z) \equiv \int_0^\infty t^{z-1} e^{-t}\, dt$

Distribution: No closed form, in general.

Triangular

Spreadsheet Function: TriangularValue

Parameters: xmin : minimum value of the returned random variable

mode : most likely value of the returned random variable

xmax : maximum value of the returned random variable

Example: = TRIANGULARVALUE(0, 1, 2)

Density:

$$f(x) = \begin{cases} \frac{2}{x_{max} - x_{min}} \frac{x - x_{min}}{c - x_{min}} & x_{min} \leqslant x \leqslant c \\ \frac{x}{x_{max} - x_{min}} \frac{x_{max} - x}{x_{max} - c} & c \leqslant x \leqslant x_{max} \end{cases}$$

Distribution:

$$F(x) = \begin{cases} \frac{(x - x_{min})^2}{(x_{max} - x_{min})(c - x_{min})} & x_{min} \leqslant x \leqslant c \\ 1 - \frac{(x_{max} - x)^2}{(x_{max} - x_{min})(x_{max} - c)} & c \leqslant x \leqslant x_{max} \end{cases}$$

Uniform

Spreadsheet Function: UniformValue

Parameters: xmin : minimum value of the returned random variable

xmax : maximum value of the returned random variable

Example: = UNIFORMVALUE(0, 1)

Density:

$$f(x)=\left\{\begin{array}{ll} \frac{1}{x_{max}-x_{min}} & x_{min}<x<x_{max} \\ 0 & otherwise \end{array}\right\}$$

Distribution:

$$F(x)=\left\{\begin{array}{ll} 0 & x<x_{min} \\ \frac{x-x_{min}}{x_{max}-x_{min}} & x_{min}<x<x_{max} \\ 1 & x_{max}<x \end{array}\right\}$$

Weibull

Spreadsheet Function: WeibullValue

Parameters: a : location parameter a, any real number

b : scale parameter b > 0

c : shape parameter c > 0

Example: = WEIBULLVALUE(0, 1, 2)

Density:

$$f(x)=\begin{cases} \frac{c}{x-a}\left(\frac{x-a}{b}\right)^{c}\exp\left[-\left(\frac{x-a}{b}\right)^{c}\right] & x>a \\ 0 & otherwise \end{cases}$$

Distribution:

$$F(x)=\begin{cases} 1-\exp\left[-\left(\frac{x-a}{b}\right)^{c}\right] & x>a \\ 0 & otherwise \end{cases}$$

Discrete Distributions

Discrete distributions return some discrete value – true or false, 0 or 1, or some other specific values – based on given probabilities and other inputs to the function.

The discrete distribution functions contained in the add-in follow. Refer to the description of each distribution for more information on the add-in function and the returned value.

BernoulliValue

BinomialValue

GeometricValue

HypergeometricValue

NegativeBinomialValue

PascalValue

PoissonValue

Bernoulli

A Bernoulli trial represents a single of a probabilistic event. The trial will either succeed (TRUE) or fail (FALSE), with the probability of success given by *p*.

Spreadsheet Function: BernoulliValue

this function is also available as *ProbabilityValue*

Parameters: p : probability of returning TRUE, where $0 \le p \le 1$

Returns: TRUE or FALSE

Example: = BERNOULLIVALUE(0.5)

Density:

$$f(k) = \begin{cases} 1-p & \text{if} \quad 0 \\ p & \text{if} \quad 1 \end{cases}$$

Distribution:

$$F(k) = \begin{cases} 1-p & \text{if} \quad 0 \leqslant k < 1 \\ p & \text{if} \quad k \geqslant 1 \end{cases}$$

Binomial

The binomial distribution represents the probability of *k* successes in *n* independent Bernoulli trials, where the probability of success in each trial is given by *p*.

Spreadsheet Function: BinomialValue

Parameters: p : probability of event p, where $0 \leq p \leq 1$

n : number of independent trials $n > 0$

Returns: the number of successful trials

Example: = BINOMIALVALUE(0.25, 10)

Density:

$$f(k) = \begin{cases} \binom{n}{k} p^k (1-p)^{n-k} & k \in \{0, 1, \cdots, n\} \\ 0 & \textit{otherwise} \end{cases}$$

Distribution:

$$F(k) = \begin{cases} \sum_{i=0}^{k} \binom{n}{i} p^i (1-p)^{n-i} & 0 \leqslant k \leqslant n \\ 1 & k > n \end{cases}$$

where the *binomial coefficient* $\binom{n}{i} = \frac{n!}{i!(n-i)!}$

Geometric

The geometric distribution represents the probability of obtaining *k* failures before obtaining a successful result in *n* independent Bernoulli trial, where the probability of success in each trial is given by *p*.

Spreadsheet Function: GeometricValue

Parameters: p : probability of event p, where 0 < p < 1

Returns: the number of failures before a successful trial

Example: = GEOMETRICVALUE(0.25)

Density:

$$f(k) = \begin{cases} p(1-p)^k & k \in \{0,1,\cdots\} \\ 0 & otherwise \end{cases}$$

Distribution:

$$F(k) = \begin{cases} 1-(1-p)^{k+1} & k \geqslant 0 \\ 0 & otherwise \end{cases}$$

Hypergeometric

The hypergeometric distribution represents the probability of *k* successes in *n* Bernoulli trials drawn without replacement (dependent) from a population *N* containing *K* successes.

Spreadsheet Function: HypergeometricValue

Parameters: n : number of trials

N : population size

K : successes contained in the population

Returns: the number of successes

Example: = HYPERGEOMETRICVALUE(6, 10, 4)

Density: $$f(k) = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$$

where $\binom{n}{k} \equiv \frac{n!}{k!(n-k)!}$ is the binomial coefficient

Distribution: $$F(k) = \left\{ \begin{array}{cc} 1-(1-p)^{k+1} & k \geqslant 0 \\ 0 & otherwise \end{array} \right\}$$

Negative Binomial

The negative binomial distribution represents the probability of *k* failures before the *s*th successful trial in a sequence of independent Bernoulli trials, where the probability of success in each trial is given by *p*.

Spreadsheet Function: NegativeBinomialValue

Parameters: p : probability of event p, where $0 \leq p \leq 1$

s : number of successes s, where $s \geq 1$

Returns: the number of failures

Example: = NEGATIVEBINOMIALVALUE(0.25, 5)

Density:
$$f(k) = \begin{cases} \frac{(s+k-1)!}{k!(s-1)!} p^s (1-p)^k & k \in \{0,1,\cdots,n\} \\ 0 & otherwise \end{cases}$$

Distribution:
$$F(k) = \begin{cases} \sum_{i=0}^{k} \frac{(s+i-1)!}{i!(s-1)!} p^k (1-p)^i & x \geqslant 0 \\ 0 & otherwise \end{cases}$$

Pascal

The pascal distribution represents the probability of having to perform *k* trials in order to achieve *s* successes in a sequence of *n* independent Bernoulli trials, where the probability of success in each trial is given by *p*.

Spreadsheet Function: PascalValue

Parameters: p : probability of event p, where $0 \le p \le 1$

s : number of successes s, where $s \ge 1$

Returns: the number of trials

Example: = PASCALVALUE(0.5, 3)

Density:

$$f(k) = \left\{ \begin{array}{cc} \frac{(k-1)!}{(k-s)!(s-1)!} p^s (1-p)^{k-s} & k \in \{s, s+1, \cdots\} \\ 0 & otherwise \end{array} \right\}$$

Distribution:

$$F(k) = \left\{ \begin{array}{cc} \sum_{i=1}^{k} \frac{(k-1)!}{(i-s)!(s-1)!} p^s (1-p)^{i-s} & k \geqslant s \\ 0 & otherwise \end{array} \right\}$$

Poisson

The Poisson distribution represents the probability of *k* events occurring where the probability of the event occurring is small, and the rate of occurrence (μ) is constant.

Spreadsheet Function: PoissonValue

Parameters: μ (mu) : rate of occurrence μ > 0

Returns: the number of events

Example: = POISSONVALUE(2)

Density:

$$f(k) = \begin{cases} \frac{\mu^k}{k!} e^{-\mu} & k \in \{0,1,\cdots\} \\ 0 & otherwise \end{cases}$$

Distribution:

$$F(k) = \begin{cases} \sum_{i=0}^{k} \frac{\mu^i}{i!} e^{-\mu} & k \geqslant 0 \\ 0 & otherwise \end{cases}$$